HW9 Solution file

Q-n 1 is 10 pt.

The computed value of F is $f = \frac{\text{MSTr}}{\text{MSE}} = \frac{2673.3}{1094.2} = 2.44$. Degrees of freedom are $I - 1 = 4$ and $I(J - 1) =$ (5)(3) = 15. From Table A.9, $F_{.05,4,15} = 3.06$ and $F_{.10,4,15} = 2.36$; since our computed value of 2.44 is between those values, it can be said that .05 < P-value < .10. Therefore, H_0 is not rejected at the $\alpha = .05$ level. The data do not provide statistically significant evidence of a difference in the mean tensile strengths of the different types of copper wires.

Q-n 3 is also 10 pt. Take off 3 pts if the null hypothesis is not stated clearly as requested.

With μ_i = true average lumen output for brand i bulbs, we wish to test $H_0 : \mu_1 = \mu_2 = \mu_3$ v. H_a: at least two μ_i's are different. $\text{MSTr} = \hat{\sigma}_B^2 = \frac{591.2}{2} = 295.60$, $\text{MSE} = \hat{\sigma}_W^2 = \frac{4773.3}{21} = 227.30$, so

$$f = \frac{\text{MSTr}}{\text{MSE}} = \frac{295.60}{227.30} = 1.30.$$

For finding the P-value, we need degrees of freedom $I - 1 = 2$ and $I(J - 1) = 21$. In the 2nd row and 21st column of Table A.9, we see that $1.30 < F_{.10,2,21} = 2.57$, so the P-value > .10. Since .10 is not < .05, we cannot reject H_0. There are no statistically significant differences in the average lumen outputs among the three brands of bulbs.

Q-n 11 is 10 pt

$Q_{.05,5,15} = 4.37$, $w = 4.37\sqrt{\frac{272.8}{4}} = 36.09$. The brands seem to divide into two groups: 1, 3, and 4; and 2 and 5; with no significant differences within each group but all between group differences are significant.

3	1	4	2	5
437.5	462.0	469.3	512.8	532.1

Q-n 12 is 10pt. Take off 3 pts in Q-n 12 if there is no clear verbal description as requested in the text of the problem.

Brands 2 and 5 do not differ significantly from one another, but both differ significantly from brands 1, 3, and 4. While brands 3 and 4 do differ significantly, there is not enough evidence to indicate a significant difference between 1 and 3 or 1 and 4.

3	1	4	2	5
427.5	462.0	469.3	512.8	532.1

Q-n 22 is 10 pt

Summary quantities are $x_{1.} = 291.4$, $x_{2.} = 221.6$, $x_{3.} = 203.4$, $x_{4.} = 227.5$, $x_{..} = 943.9$, $CF = 49,497.07$, $\Sigma\Sigma x_{ij}^2 = 50,078.07$, from which SST = 581,

$$\text{SSTr} = \frac{(291.4)^2}{5} + \frac{(221.6)^2}{4} + \frac{(203.4)^2}{4} + \frac{(227.5)^2}{5} - 49,497.07 = 49,953.57 - 49,497.07 = 456.50 \text{, and}$$

SSE = 124.50. Thus $\text{MSTr} = \frac{456.50}{3} = 152.17$, $\text{MSE} = \frac{124.50}{18-4} = 8.89$, and f = 17.12. Because $17.12 > F_{.001,3,14} = 9.73$, P-value < .001 and $H_0 : \mu_1 = ... = \mu_4$ is rejected at level .05. There is a difference in true average yield of tomatoes for the four different levels of salinity.

Q-n 24 is 10 pt; 5 pts for the basic F-test and 5 pts for the Tukey procedure.

Let μ_i denote the true average skeletal-muscle activity the ith group (i = 1, 2, 3). The hypotheses are H_0: $\mu_1 = \mu_2 = \mu_3$ versus H_a: at least two of the μ_i's are different.
From the summary information provided, $\bar{x}_{..} = 51.10$, from which

$$\text{SSTr} = \sum_{i=1}^{3}\sum_{j=1}^{J_i}(\bar{x}_{i.} - \bar{x}_{..})^2 = \sum_{i=1}^{3} J_i(\bar{x}_{i.} - \bar{x}_{..})^2 = 797.1. \text{ Also, } \text{SSE} = \sum_{i=1}^{3}\sum_{j=1}^{J_i}(\bar{x}_{ij} - \bar{x}_{i.})^2 = \sum_{i=1}^{3}(J_i - 1)s_i^2 = 1319.7.$$

The numerator and denominator df are $I - 1 = 2$ and $n - I = 28 - 3 = 25$, from which the F statistic is

$$f = \frac{\text{MSTr}}{\text{MSE}} = \frac{791.1/2}{1319.7/25} = 7.55.$$

Since $F_{.01,2,25} = 5.57$ and $F_{.001,2,25} = 9.22$, the P-value for this hypothesis test is between .01 and .001. There is strong evidence to suggest the population mean skeletal-muscle activity for these three groups is not the same.

To compare a group of size 10 to a group of size 8, Tukey's "honestly significant difference" at the .05 level is $w = Q_{.05,3,25}\sqrt{\frac{\text{MSE}}{2}\left(\frac{1}{10} + \frac{1}{8}\right)} \approx 3.53\sqrt{\frac{52.8}{2}\left(\frac{1}{10} + \frac{1}{8}\right)} = 8.60$. So, the "old, active" group has a significantly higher mean s-m activity than the other two groups, but young and old, sedentary populations are not significantly different in this regard.

Young	Old sedentary	Old active
46.68	47.71	58.24